2/1/1991

HOW SHOULD WE USE ENTROPY IN ECONOMICS?

(Some half–baked ideas in need of criticism)

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Abstract. Classical economics was built largely on the analogy to mechanics, as it was known in the time of Adam Smith; particularly the idea of mechanical equilibrium. But a macroeconomic system is in some ways more like a thermodynamic system than a mechanical one, so we develop that analogy. Since the time of J. Willard Gibbs it has been known that prediction of chemical processes – reversible or irreversible – could not possibly have succeeded until the entropy of a macrostate was recognized and taken into account. We conjecture that the same may be true in economics; the direction of economic change may have as much to do with the entropies of neighboring macrostates as with any of the other 'dynamical' factors now recognized.

Crises in Physics and Economics

Physics is by far the oldest of the quantitative sciences, and so it is hardly surprising that some of the conceptual and methodological problems arising in newer sciences have turned out to be almost identical with ones that were recognized and solved long ago in physics.

As we read in the newspapers, both Keynesian and Monetarist economic theories have been unsuccessful in accounting for recent economic behavior. Neither can point to particularly great past successes from adoption of their policies by Government, and – if we can believe the press – neither seems to have any clear idea of what Government should be doing now.

Of course, this does not mean that all their equations are wrong; many express obvious conservation laws that no sane person can doubt. It does mean that both systems are incomplete – if neither accounts for actual behavior, then it follows that actual behavior is being determined by some factor that neither has yet recognized.

Physics has been in this methodological crisis many times. We invent a theory describing how certain variables $(X, Y, Z \cdots)$ should be related; but Nature persists in behaving differently. Like the economists, we try first to patch things up by trying out different functional relations (in physics, different masses and force laws, *etc.*; in economics, different multipliers, consumption functions, *etc.*).

But eventually we are forced to recognize that Nature is not obeying any laws that are expressible in terms of $(X, Y, Z \cdots)$. Then we have a methodological crisis: our problem is not just quantitative. Our theory is qualitatively wrong, because we have chosen the wrong set of variables. At this point, no amount of computing power, no amount of mathematical skill in manipulating the old variables, can help us – only new *ideas*.

At the turn of the Century we learned that the real world of physics is not describable merely in terms of particles interacting via central forces, in imitation of Newtonian cosmology. This was the methodological crisis that Albert Einstein and Max Planck faced; their conceptual innovations started our present relativity and quantum theories.

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Today, it appears that the real world of economics might not be describable merely in terms of conventional macroeconomic variables (unemployment rate, GNP, aggregate demand, *etc.*) If so, then a conceptual innovation is called for. So what is the missing factor?

An economic system is in some ways like a mechanism, as is recognized in all theories. But it is really more like a thermodynamic system than a mechanism – an analogy also noted by others, but not yet developed sufficiently to judge the possibilities.

In the following we present some very tentative, preliminary conjectures about what an economic theory based on the thermodynamic analogy rather than the mechanical one might look like. Although these ideas are in need of much criticism, and no doubt revision, some interesting – and to the writer quite plausible – new structure appears.

The Economic – Thermodynamic Analogy

On this analogy, the failure of Keynesian and Monetarist mechanisms to account for recent economic behavior would be attributed, at least in part, to their failure to recognize the entropy factors that must ultimately control economic change and equilibrium, just as they do in thermodynamics.

That is, it may be that a macroeconomic system does not move in response to (or at least not solely in response to) the "forces" that are supposed to exist in current theories; it may simply move in the direction of increasing entropy as constrained by the conservation laws imposed by Nature and Government – just as a thermodynamic system makes its approach to equilibrium in the direction of increasing entropy as constrained by the conservation of mass, energy, etc.

In physics, the thermodynamic entropy of a macrostate (defined by specifying pressure, volume, energy, etc.) is essentially the logarithm of the number of microstates (quantum states) consistent with it; *i.e.*, the number of ways the macrostate can be realized.

Likewise, the "economic entropy" S to which we refer is a function

$$S(X, Y, Z \cdots) = \log W(X, Y, Z \cdots)$$

of whatever macroeconomic variables $(X, Y, Z \cdots)$ our theory recognizes. Here W is the multiplicity factor of the macroeconomic state (number of different microeconomic ways in which it can be realized).

In a probabilistic model of the economy, we ought to include in the probability of any macroeconomic state an entropy factor $\exp(S)$ to take this multiplicity into account. This is one of the factors, possibly the only variable factor, in the prior probability of that state. If we failed to do this in statistical mechanics – particularly in chemical thermodynamics – we would get grossly, qualitatively wrong predictions, and the same may be true in macroeconomics.

Of course, merely to conjecture this does not prove that entropy is the crucial missing factor; it may be that some other unrecognized factor is even more important. But entropy is at least a promising candidate, because it is clearly relevant, and it is not now being taken into account (we might add that entropy is completely non–ideological, having nothing to do with any social philosophy; and so the idea ought to be equally acceptable to all).

The physical analogy can help us much more than this. At what velocity does the economic system drift up the entropy hill? How widely will it fluctuate about the deterministic path? The answers were first seen intuitively in early work of Einstein, Fokker–Planck, and Onsager. Today they are all subsumed in the general formalism of Predictive Statistical Mechanics (Jaynes, 1980, 1985), in which the equilibrium maximum–entropy variational principle of Gibbs is generalized to time–varying phenomena.

What that theory suggests is the following. Even though a neighboring macroeconomic state of higher entropy is available, the system does not necessarily move to it. A pile of sand does not necessarily level itself unless there is an earthquake to shake it up a little. The economic system might just stagnate where it is, unless it is shaken up by what an Englishman might call a "dither" of some sort.

Of course, stagnation is not necessarily bad in itself; stagnation at a point where everybody is happy might even be perceived as the goal of economics. But in the past, stagnation seems to have occurred at points where almost everybody was unhappy, and wanted a change (as political slogans of the genre: "Let's get the country moving again!" testify).

The Dither

In our conjectured picture of things, the dither that prevents economic stagnation and drives us up the entropy hill is a kind of turbulence injected into the macroeconomic variables by fluctuations in the underlying microeconomy. By this means, the macroeconomic state is constantly driven to "exploring the possibilities" of neighboring states.

There is a close analogy in the mechanism of biological evolution. There the dither that drives the process is spontaneous random mutations, as a result of which every species is constantly exploring the possibilities of a slightly different design. A "good" mutation has a better chance than a "bad" one of reproducing itself, and therefore becomes gradually more representative of (*i.e.*, a larger portion of) the species.

Like molecules of a gas which in their motion eventually explore every niche and crevice of the volume available to them, mutations would eventually "try out" every conceivable form of a creature. But the biological process is very much slower, and we think that in the age of the universe, biological variation has explored only a negligibly small part of all possibilities. Presumably, the human race has thus far explored only a negligibly small portion of all the possibilities in economics.

In economics, the idea of the dither was anticipated by Keynes, who attributed it to "animal spirits", which cause people to behave erratically. We think of the dither in more general terms, simply the result of many independent individual decisions, not necessarily erratic or irrational. Indeed, most individuals will act according to what seem to them, "rational expectations."

However, without the entropy factor Keynes did not find the phenomenon that our model considers a fundamental cause of economic change. In constantly exploring the neighboring states, the economy is always more likely to move to one of higher than lower entropy, simply because there are more of them (greater multiplicity). Thus the dither not only introduces random uncertainty into macroeconomic variables, *it drives a systematic movement of the economy.* In fact, mathematical analysis shows that the average drift velocity in the macroeconomic space is proportional to:

$$(\text{entropy gradient}) \times (\text{mean} - \text{square fluctuation}). \tag{1}$$

Thus we see that economic stagnation can have two quite different causes: loss of entropy gradient, and loss of dither. Without an entropy gradient, the sense of direction is lost and the system drifts aimlessly. Usually one calls this motion 'random' but what we really mean by that is 'determined not by any macroeconomic variables but by unrecorded details of microeconomic variables'. When the Government changes policies, it is changing the entropy function or the domain in which that function can exist (the "support" of the function). When individual citizens become poor or prosperous, pessimistic or confident, cautious or adventuresome, it changes the dither.

On this picture Government policy may be thought of as having two different sides; "hard" policy like bank reserve requirements that place rigid boundaries on the accessible points of the space, and "soft" policy like changing price support levels or tax rates, that tilt the entropy function to steer the system, so it moves of its own accord – hopefully in the right direction and at a safe velocity. To guide that policy econometrics would, among other things, try to determine the current entropy gradient and dither, from theory and the available data.

The ideas of "steering" and "fine–tuning" have of course been with us for a long time, although not very successfully and not in connection with achieving a desired entropy function.

Bubble Dynamics and Catastrophe Theory

The further quantitative development of this line of thought (Jaynes, 1985) is what we have called "bubble dynamics". We have a bubble of probability in the macroeconomic space, whose center of gravity follows a deterministic path up the entropy hill, but whose size and shape constantly change in adjustment to the local curvature of the entropy function. That is, the dither generates a random walk in the macroeconomic space, which is then steered and stabilized by the entropy function.

We find that deterministic, random, and unstable behavior are all exhibited by this model, as follows. A strongly convex entropy function is strongly stabilizing, leading to a small bubble; *i.e.* nearly deterministic behavior. When the curvature of the entropy function decreases, stabilizing forces are weaker and the bubble enlarges, representing more "random" behavior (by which we really mean "less predictable from macroeconomic variables").

When the entropy curvature is zero (*i.e.* entropy is a locally linear function), the restoring forces are zero, and the bubble spreads following Einstein's law of Brownian motion (dimensions growing as the square root of the time).

When convexity of the entropy fails altogether (*i.e.* its curvature is negative) we have instability, perhaps a political revolution, which is mathematically a bifurcation, the bubble stretching out and splitting into two smaller bubbles that go their separate ways, each representing a different possible society growing out of the revolution. So if these conjectures should prove correct, Governments of the distant future will keep an eye not only on the local tilt of the entropy function, but also on its local curvature.

Some of these points are anticipated in Rene Thom's catastrophe theory; however in applications to economic and/or political stability, it has not been clear which quantity should be thought of as having these convexity–topological properties; *i.e.* to what function one should apply these considerations? We suggest that it is the entropy function – for then the stability criteria are just the convexity properties given by Gibbs (1873) in his analysis of thermodynamic stability. Analytical solutions displaying all these features have been found; details will be given elsewhere.

Indeed, Thom's fold catastrophe (a response variable $z(x, y)$ determined by $y = z^3 + xz$) is mathematically identical with the thermodynamics of a ferromagnet, standard in physics for about seventy years. Here y = magnetic field H, z = magnetization M, $x = T - T_c$, where T = temperature, T_c = Curie critical temperature. The resulting "catastrophe" (spontaneous magnetization at temperatures below the Curie temperature) makes possible permanent magnets and causes the "B-H hysteresis loops" familiar to Freshman physics students.

Gibbs' original explanation of such catastrophes in terms of entropy convexity is simpler and easier to visualize than Thom's multiple–valued folded surfaces. Introducing the entropy function $S = -(1/4)(x + z^2)^2$, Thom's response function looks like a standard thermodynamic equation: $y = -dS/dz$. But in Gibbs' choice of variables there is no multiple valuedness. The catastrophe is now seen merely as the consequence of a dimple (local loss of convexity with respect to z) that develops in the entropy function when $x < 0$.

Gibbs explained all the phase transitions that occur in multidimensional thermodynamic spaces as "catastrophes" arising from the various kinds of local loss of convexity that can occur in an entropy function.

Thom's swallowtail catastrophe ($y = z^5 + wz^3 + uz^2 + vz$) is generated by the variety of dimples that can develop in the sixth degree entropy function

$$S = -(1/6)(u + z^3)^2 - (1/2)vz^2 - (1/4)wz^4 \tag{2}$$

as (u, v, w) vary. The extremely complicated butterfly catastrophe is just the result of the dimples that can appear in an entropy function of seventh degree, and so on. Although not all details are yet written down, we expect that all of Thom's catastrophe forms can be generated and understood, without multiple–valuedness, in Gibbsian terms of entropy convexity, and in that respect are present automatically in our conjectured form of economic theory.

However, as the name implies, bubble dynamics has more content than catastrophe theory, which describes only the equilibrium states and not the dynamics telling us along what path and at what velocity the system gets to those states. Bubble dynamics can, for example, describe the fine details of time development at a bifurcation point, determining what fraction of the bubble will move to the left or right (*i.e.*, given the size, shape, and position of a bubble approaching a bifurcation point, what is the probability that the system will move ultimately to the left or right?).

Conclusion

To apply these ideas to real economic prediction would require the kind of judgment that comes from long familiarity with the subject–matter. One needs to know which particular macroeconomic variables should be included to have a "full set"; and how to define the underlying microeconomic "hypothesis space" that determines the multiplicity factors. On such matters the writer is willing to hazard some guesses, but feels the need of help. Presumably, a process of trial and error will be needed to find the right choices. Therefore we think that a realistic implementation of this thermodynamic analogy still lies rather far in the future.

Although we have, for expository purposes, stressed only the ways in which our conjectured theory would differ from current ones, we do not contemplate that very much of existing theories would be lost in the process of adding these new features; rather their successful parts would be incorporated into the new theory. Established truths do not lose their validity when new truths are added to them.

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